Probabilistic Concept of Spark Ignition of Fuel Droplets Suspended in Air

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ABSTRACT

A new probabilistic concept of spark ignition of fuel droplet suspension in air at fuel-air ratio near lean flammability limit is presented. The probability of spark-ignition of heterogeneous mixture (with very low concentration of evaporated phase and high concentration of liquid phase) is caused by two main facts:
a) combustible concentration in air is non-continuous,
b) energy production in the spark and its transport in the mixture depends on time. Due to these, droplets may be situated outside high energy region and the ignition is impossible.

Though probabilistic feature of spark-ignition has been observed for homogeneous mixtures, only a few author observed and measured it for heterogeneous mixtures. In this paper two mechanisms of spark discharge are analysed: hydrodynamic and thermal. The latter is proposed and used to formulate the theory of spark-ignition that enables computation of probability ignition and minimum ignition energy.

INTRODUCTION

In the last decade spark ignition of gaseous mixtures has been the problem of many approaches. The probabilistic feature of this phenomenon has been observed in the case of homogeneous mixtures by a few authors [1-6]; in the literature a minimum ignition energy is used to denote spark energy level at 50% of probability [1-5]. Also a great number of papers has been published on modelling of spark-ignition in which necessary conditions for ignition (ignition criteria) have been formulated, with which evaluation of the influence of mixture parameters on ignition phenomenon is possible and which enables computation of minimum ignition energy [6-9]. However, neither the publications on theory of spark ignition of heterogeneous mixtures (in which fuel is in liquid state as droplets) nor any data on probability of ignition of these mixtures can be found (except the study of Kumagai who observed probabilistic feature of self - ignition of droplets injected into hot air [10]). Problem of spark-ignition is very serious from the point of view of non-uniform engine operation, losses of engine power and efficiency and hydrocarbon emission caused by misfires, especially near lean flammability limit of the mixture.

In this paper a probabilistic theory of spark ignition of heterogeneous mixture based on the model of the spark discharge and the model of the mixture is presented.

PROBABILISTIC CONCEPT OF SPARK IGNITION

Formulation of the Problem

Near lean limit of flammability of fuel droplet suspension, the probability of spark ignition may be defined as probability of conditional phenomena

$$P = P(a)\,P(b/a) \tag{1}$$

where:
P(a) - probability of spark ignition of a single (first) drop
P(b/a) - conditional probability of ignition of another (second) drop by the first one, in its close vicinity

Values of each above mentioned probabilities lie in the range (0,1).

In the case of $P(a) < 1$ spark energy is too small to ensure successful ignition by a single spark discharge and always $P < 1$.

In the case of $P(b/a) < 1$ mixture is in general - inflammable (if $\alpha < 1$ the distance between droplets is too large and if $\alpha \geq 1$ fuel - air ratio of the mixture may be below flammability limit or/and the distance between the droplets is too large). If $P(b/a) = 1$ and $P(a) < 1$, probability of ignition P_n^m by n successive spark is determined by the binomial distribution function

$$P_n^m = \frac{n!}{m!(n-m)!} P^n (1-P)^{n-m} \tag{2}$$

where P is the probability of single trial (single spark), n is the number of successive spark discharges and m is the number of successful ignitions of the mixture.

Probability P(a) is assumed to be equal to the ratio of the diameter Δx_1 of the volume of space in which a single drop can be ignited by the spark (i.e. its residence time in the volume, in which temperature is higher than ignition temperature, is longer than its ignition delay) to the distance between droplets suspended in air. Similarly P(b/a) is assumed to be equal to the ratio of the maximum distance Δx_2 between burning droplet, that just has been ignited by the spark, and the second one in its close vicinity, which may be ignited by the flame attached to the first burning droplet to the distance between droplets. Value of Δx_1 and Δx_2 depends on the kind of heat source modelling the spark discharge and on the diameter of burning droplet, respectively. The manner of computation of Δx_1 and Δ_2 is given below. In the general case the distance between droplets is statistical quantity of which density of probability may be assumed. Similarly, droplet diameter may by also non-uniform, its size may by described by any distribution function, say by Rosin-Rammler distribution function. However, for simplicity the distance between droplets may by obtained under the assumption that:

- all droplets have uniform diameter $(d = d_S)$
- droplets create rectangular network in which the distance between droplets is L (Fig.1).

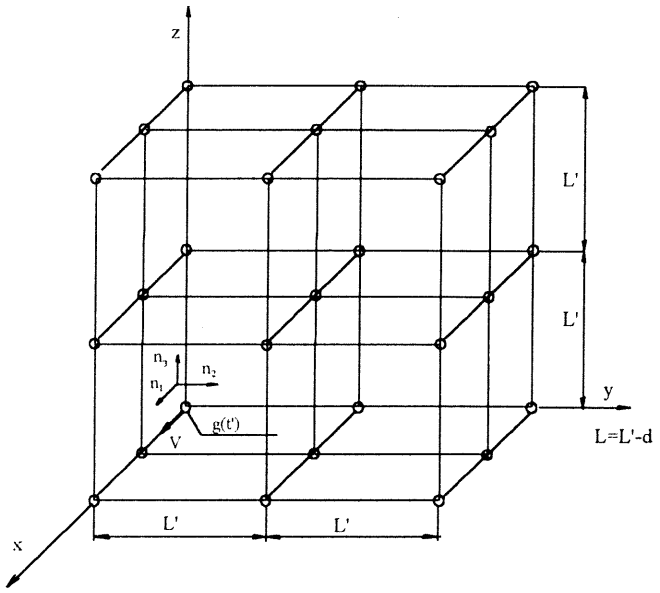

Fig.1. Rectangular network of droplets

According to this:

$$P(a) = \frac{\Delta x_1}{L\sqrt{3}} \tag{3}$$

and

$$P(b/a) = \frac{\Delta x_2}{L} \tag{4}$$

where

$$L = d\sqrt[3]{\frac{\pi}{6}\,\alpha L_o\,\frac{\rho_f}{\rho}} - 1 \tag{5}$$

DISCUSSION OF SPARK IGNITION MECHANISM OF A SINGLE DROPLET

Remarks on Spark Kernel Development

Schlieren photographs of spark kernel development revealed existence of a shock wave in early stage of kernel growth [11-13], which rapidly decays in a few microseconds. This phenomenon may be simulated by spherical explosion. Within a spherical volume behind a shock wave the temperature may be high enough during a period of time not shorter than ignition delay of a droplet so that it may be ignited. The blast wave may also disintegrate larger droplets [14]. During spark discharge an energy is liberated as a heat. Therefore spark discharge may be also simulated by a heat source, from which heat is transported to the gas due to conductivity. Also in this model, there is a spherical volume in which the temperature in high enough during a period of time not shorter than ignition delay of droplet, so that it may by ignited. If spark kernel reaches a critical radius, it may develop further and is not quenched [15].

The first model of spark discharge is of gas-dynamic nature, while the second one-thermal. Both models are used below to determine necessary conditions of droplet ignition.

Gas-dynamic Mechanism of Droplet Ignition

Spark discharge is simulated by spherical explosion of which radius and after which temperature and velocity may be expressed by Sedov's formula [16]:

$$r = \left(\frac{\beta Q}{\rho_1}\right)^{\frac{1}{5}} t^{\frac{2}{5}} \tag{6a}$$

$$T_2 = \frac{8}{25}\,\frac{k-1}{(k+1)^2}\left(\frac{\beta Q}{\rho_1}\right)^{\frac{2}{5}}\frac{t^{\frac{6}{5}}}{R} \tag{6b}$$

$$V_2 = \frac{4}{5(k+1)}\left(\frac{\beta Q}{\rho_1}\right)^{\frac{1}{5}} t^{\frac{3}{5}} \qquad (6c)$$

where $\beta = 2$ [16].
Ignition delay is expressed by Arrhenius function

$$\tau_{ign} = C \exp(\varepsilon / RT) \qquad (7)$$

In order to obtain explicit solution, one puts $T = 0.5\ T_{ign}$, and after some further simplifying assumptions, bearing in mind that droplet time Δt should be longer than ignition delay, $\Delta t > \tau_{ign}$, the radius of the sphere within which a droplet may be ignited is given by [13]:

$$r_{ign} = 0.326 \cdot \sqrt[3]{\frac{Q}{\rho_1 R T_{ign}}} \qquad (8)$$

Thermal Mechanism of droplet Ignition

Spark discharge is simulated by a point-like momentary heat source, heat is transported due to conductivity. Temperature distribution is given by [17]:

$$T = T_o + \frac{Q}{8\,\rho c_p (\pi a t)^{3/2}} \exp\left(-\frac{r^2}{4at}\right) \qquad (9)$$

The maximum radius of a sphere($r = \sqrt{6\,at}$ at a moment t) at which $T(r) = T_{ign} = \max$ is determined from Eq.(9) by the condition (Fig.2):

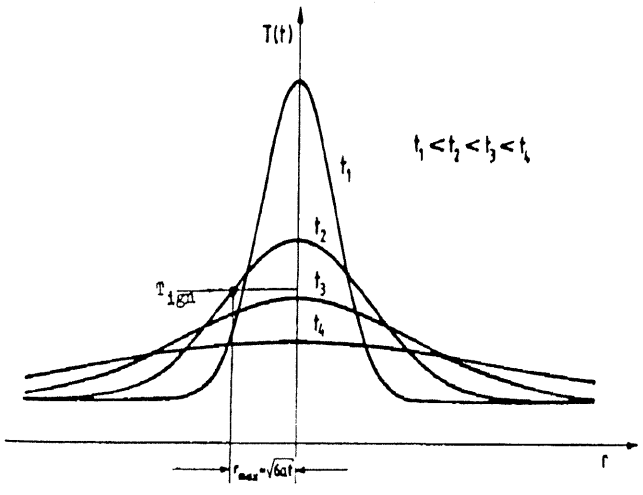

Fig.2. Temperature distribution for point like momentary heat source and maximal radius of a sphere r_{max} within which $T > T_{ign}$

$$\left(\frac{\partial r}{\partial t}\right)_{T=T_{ign}} = 0\ , \quad \left(\frac{\partial T}{\partial t}\right)_{r=const} = 0 \qquad (10)$$

is:

$$r_{max} = 0.415 \cdot \sqrt[3]{\frac{Q}{\rho c_p \Delta T_{ign}}} \qquad (11)$$

where $\Delta T_{ign} = T_{ign} = T_o$.

Comparison of Both Mechanisms

As may by noticed, the radius of spherical region in which droplet may by ignited for both mechanism: gas-dynamic and thermal is expressed by similar equations: (8) and (11), although values of T_{ign} aren't the same.
Detailed analysis of both mechanism leads to the following conclusions:

- radii of the sphere in which droplet ignition is possible for both mechanism are of the same order for the same released energy Q
- according to gas-dynamic model, droplet is disintegrated much more earlier then can be ignited and the rate energy transport is much more higher then in thermal mechanism, so droplet may miss its ignition.

For further theoretical investigations thermal mechanism of droplet ignition is assumed.

IGNITION OF SINGLE DROPLET

According to thermal mechanism droplet may be ignited within the spherical region of the radius given by Eq.(11), so $\Delta x_1 = r_{max}$.
Non-steady ignition temperature obtained under the assumptions: $T = 0.5\ T_{ign}$ and that residence time

$$\Delta t \geq \tau_{ign} = C \cdot \exp\left(\frac{\varepsilon}{RT}\right) \qquad (12)$$

is given by:

$$T_{ign} = \frac{2\varepsilon / R}{2\ln L - \ln(A/8a)} - T_o \qquad (13)$$

Ignition energy may be computed from the condition a $P(b) = 1$ with the use of Eqs (11) and (13), putting $E = Q$.

$$E \approx 0.85\ L^3 \rho c_p\ \Delta T_{ign} \qquad (14)$$

IGNITION OF DROPLET BY ANOTHER BURNING DROPLET

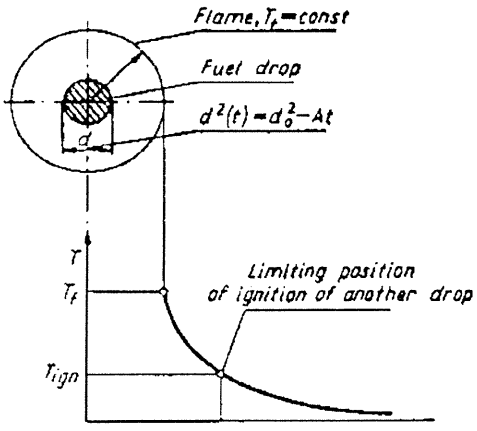

Fig.3. Temperature distribution around burning droplet

In this case an ignited and burning droplet is being the source of heat for ignition of the other droplet in its vicinity (Fig.3). The ignition criterion is equation (12). A simplified assumptions is made : T(t) in Eq.(12) is constant as follows:

$$\overline{T} = \frac{1}{\Delta t}\int_0^{\Delta t} T(t) = \frac{1}{2} T_{ign}$$

Following main assumption are made to solve that problem of ignition:

- instantaneous burning droplet diameter d(t) fulfils the equation [18]:

$$d^2(t) = d_0^2 - K\,t \qquad (15)$$

- flame radius is proportional to burning droplet diameter r_f [19]:

$$r_f = B\,d \qquad (16)$$

In order to get explicit solution for r it is assumed that during burning time, droplet diameter in Eq.(15) is constant:

$$\overline{d} = \frac{1}{\tau_0}\int_0^{\tau_0} d(t)\,dt = \frac{2}{3} d_0 \qquad (17)$$

Then one obtains the equation:

$$r = \frac{2Bd_oT_f}{3(T_{ign} - T_o)}\,\mathrm{erfc}\left(\frac{r - r_f}{\sqrt{4a\tau_o}}\right) \qquad (18)$$

From Eq.(18) $r = r_{max}$ at which $T = T_{max} = T_{ign}$ may be computed of course, $r_{max} = \Delta x_2$ in Eq.(4).

EXPERIMENTAL VALIDATION

Experimental investigations of the probability of spark-ignition of kerosene droplets suspended in air stream have been carried out [20]. In the experiments, the value of probability of spark-ignition was assumed as the ratio of the number of successful ignitions of the mixture to the assumed number of the trial of independent discharges, N, where N=20.

In a result of investigations the influence of mixture parameters and spark energy on ignition probability has been examined - Fig.4 and Fig.5. Also minimum ignition energy has been measured. Computed and experimental values of ignition probability are given in Table 1.

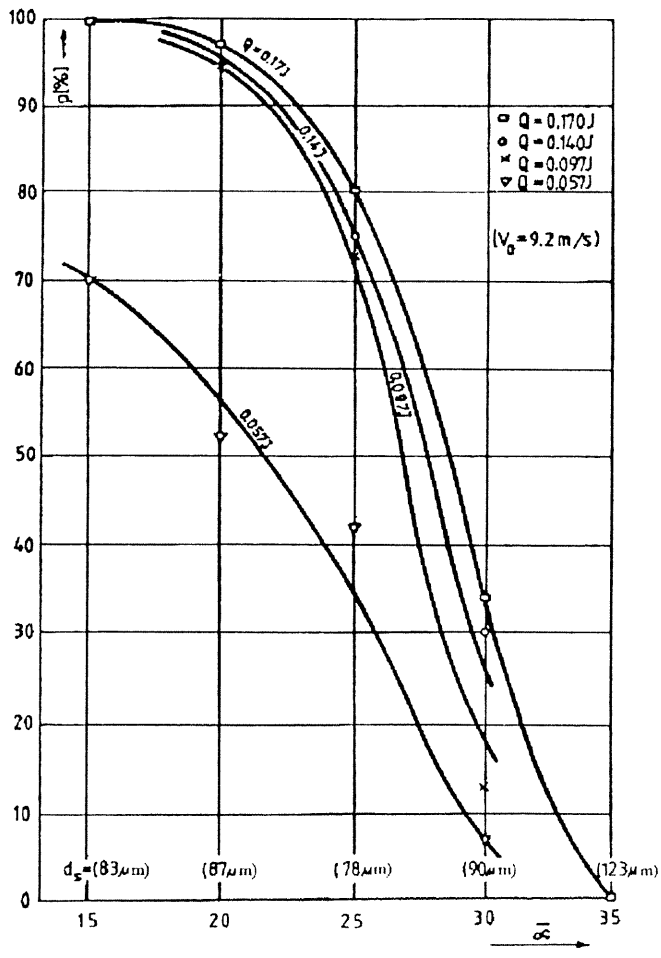

Fig.4. Ignition probability vs Sauter mean diameter d_S and mean air excess ratio α (of entire spray) for different spark energies

TABLE 1

Equivalence ratio	3.0	2.5	2.0	1.5
Measured for Q = 0.097 J d_s µm	0.07 (90)	0.42 (78)	0.52 (87)	0.7 (83)
Computed for Q = 0.097 J d_s = 100 µm	0.8	0.86	0.92	0.99

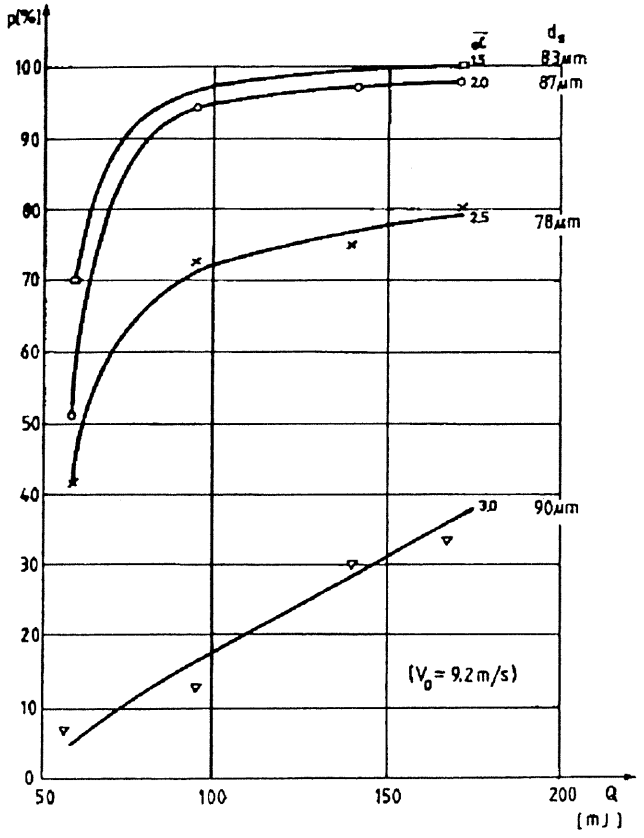

Fig.5. Ignition probability vs spark energy E for different Sauter mean diameters

Computed probability values are much more higher than measured, but the direction of changes is the same.

CONCLUSIONS

The above mentioned probability concept enables evaluation on spark ignition of fuel droplets:
- Sauter mean droplet diameter, d_s
- fuel concentration
- spark energy, E.

It enables also understanding of spark ignition mechanism and observed experimental phenomena. The further more accurate treatment of ignition mechanism within this concept is possible.

NOMENCLATURE

A - constant in Eq.(13) and (9)

$$\left(A = \frac{1}{8\,\rho\, c_p\, (\pi a)^{3/2}} \right)$$

a - thermal diffusivity

$$\left(a = \frac{\lambda}{\rho\, c_p} = 22.2 \cdot 10^{-6}\ m^2 s^{-1} \right)$$

B - constant in Eq.(16) (B = 9 [19])
C - constant in Eq.(12) (C = 0.935 10^{-11}s [21])
c_p - specific heat of gaseous phase
d - droplet diameter
E - spark energy
K - vaporisation/combustion constant in Eq.(16) $\left(K = 95 \cdot 10^{-8}\ m^2 s^{-1}\right)$
k - adiabatic exponent (k = 1.4)
L - droplet distance in rectangular network (Fig.1)
L_o - mixture stoichiometric air-fuel ratio (L_o= 14.7 kg air/kg fuel)
P - probability
Q - energy (heat) of spherical explosion (point like momentary heat source)
R - gas constant
r - radius
T - temperature
Δt - residence time of a droplet
T_f - flame temperature (T_f = 1850°C)
α - mixture air excess coefficient

$$\left(\alpha = \frac{\text{mass of the air}}{\text{mass of the fuel } L_o} \right)$$

ε - activation energy (ε/R = 23300 K [22])
λ - heat conductivity
ρ - density
τ_{ign} - ignition delay

SUBSCRIPTS

f - flame
ign - ignition
S - Sauter mean

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