

# Heat Transfer Analysis through Heterodyne Holographic Interferometry

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## ABSTRACT

The basic problems of quantitative determination of heat transfer measurements through holographic interferometry are discussed. A solution based on the measurements of local derivatives of the holographically recorded gas temperature fields is described. This novel method uses essentially heterodyne holographic interferometry for the quantitative determination of the gas temperature from the fringe pattern in the image plane. With this technique the interference phase can be measured with an accuracy and reproducibility of  $\pm 0.3^\circ$  at any point in the fringe pattern, corresponding to an interpolation of better than  $10^{-3}$  of one fringe with a high spatial resolution.

A theoretical method has been developed to predict the temperature profiles in the boundary layer as function of the heat source distribution and the physical properties of the plate of finite thickness with an arbitrary temperature distribution on its surface.

Experimental verification of the numerical results is provided by heterodyne holographic measurements of the gas temperatures, boundary layer thickness and boundary layer gas temperature distribution on heated horizontal plates. The results are rather preliminary, therefore, further computational work is necessary.

## INTRODUCTION

The general objectives have been experimental and theoretical investigations of methods for the quantitative determination of temperature profiles in boundary layers from holographic interferometry. This includes the development of special opto-electronic systems to increase the sensitivity and the accuracy for the interpolation of holographic interference patterns as well as the development of analytical and numerical methods to calculate the temperature fields.

The basic principle of holography was invented and described by D. Gabor [1] in 1948. Holography can be essentially described as an optical technique, which allows to store the entire information contained in the optical wavefield and to reconstruct this same wavefield at any desired later time. Therefore holography is rather an intermediate, self-reproducible step than an actual processing of in-

formation in optical arrangements.

The discussion about the dimensionality of the information content of a hologram is relatively old [2, 3]. It has been treated conclusively by several authors [4, 5] and has become meanwhile one of the *Paradoxa aus der Optik* [6]. The answer is simply that a hologram does not contain the complete information about a true three-dimensional object. But this is rather a consequence of the basic propagation properties of the optical wavefield than of the limitations of a hologram. A hologram can store and reconstruct completely the complex amplitude of a polarized wavefield, except for an arbitrary absolute phase, at least within the limits due to size and resolution of the holographic recording material. Concerning the dimensionality of an object one should strictly distinguish between true three-dimensional, optically transparent objects, which are described by a three-dimensional distribution of the refractive index  $n(x,y,z)$ , and solid object in three-dimensional space, which are optically determined by their two-dimensional, diffusely scattering surface  $z = f(x,y)$ . In the following the possibilities for the reconstruction of transparent objects, i.e. the quantitative determination of their internal three-dimensional structure, from holographically recorded information will be discussed.

The inverse scattering method, based on the first Born approximation, is only applicable to weakly scattering objects, described by a complex index of refraction  $n(x,y,z)$  [3,4,5]. It has been shown [4] that a single direction of illumination does only give two-dimensional information, and that therefore a set of observations with different directions of illumination have to be used for the complete reconstruction of the three-dimensional refractive index  $n(x,y,z)$ . The first Born approximation requires that the intensity of the wave scattered by the object structure  $F(x,y,z) = n(x,y,z) - n_0$  remains small compared with the intensity of the illuminating wave. This is in the case of realistic phase objects a very severe limitation and therefore the experimental verification of the inverse scattering method has not been very successful so far [7,8].

For the reconstruction of phase objects described by a real and continuous refractive index  $n(x,y,z)$  the straight line approximation [9] turns out to be a more suitable method. This method requires only that the average gradient of the refractive index is small enough so that the light paths through the object are straight lines within the considered

spatial resolution. This condition is much less restrictive than the requirements for the inverse scattering method in phase objects. Also in the straight line approximation a set of observations under different directions have to be used for the complete reconstruction of the three-dimensional refractive index  $n(x,y,z)$ , since the information obtained from a single observation is only two-dimensional. In practical applications, serious problems occur in the numerical evaluation due to the nonregular distribution and the limited accuracy of the data samples obtained from the interferograms [10]. Several different computational techniques have been developed and tested to achieve the most accurate reconstructions from discrete data collected over a limited range of ray directions [10,11]. Although the straight line method has been successfully applied to the determination of the three-dimensional temperature field in interacting thermal plumes above two heaters submerged in water [12], the fringe counting was the least accurate of all measurements and imposed the major limitations to the overall accuracy of the experiment. The experimental difficulties are illustrated by the fact that for the reconstruction of the temperature field in 27 space points as much as 934 fringe positions at 17 different angles of observation over a range of  $46^\circ$  had to be measured manually. The uncertainty in the reconstructed temperature was found to be  $\pm 10\%$  with respect to the maximum temperature difference of about 10 K, which corresponds to a resolution for the refractive index of about  $\Delta n = 10^{-4}$  in water.

The conclusions are that there exist useful and quite well developed methods to reconstruct three-dimensional phase objects from holographic interferometric data, but that a faster and more accurate technique for fringe evaluation is strongly needed.

Heterodyne holographic interferometry will be the solution for this remaining experimental problem.

### Holographic Interferometry

Interferometry is an old and very powerful technique to measure the deviations between two wavefields with a sensitivity of a fraction of a wavelength. Coherent light  $V(\vec{x},t)$  of high optical quality mostly plane or spherical waves, probes the object by reflection or transmission. This results in a wavefield

$$V'(\vec{x},t) = a(\vec{x}) e^{i\phi(\vec{x})} V(\vec{x},t) \quad (1)$$

with slightly distorted phase  $\phi(\vec{x})$  and amplitude  $a(\vec{x})$ . It is mainly the distribution of the phase  $\phi(x,y)$  in a plane  $(x,y)$  near the object, which carries the wanted information. That information is extracted by superposition of  $V(\vec{x},t)$  and  $V'(\vec{x},t)$  with the help of some kind of a beam splitter and looking at the resulting intensity

$$\begin{aligned} I(x,t) &= |V+V'|^2 \\ &= |V|^2 \left[ (1+a^2) + a(x,y) \cos\phi(x,y) \right] \end{aligned} \quad (2)$$

in the image of the considered plane near the object. The result is an interference fringe pattern, where the phase information  $\phi(x,y)$  is transformed by the cosine-function into an intensity distribution. As long as the local amplitude variations  $a(x,y)$  are small compared with the fringe separation the phase

information can be quantitatively extracted from the maxima and minima of the fringes and sometimes even by carefully interpolating between the fringes.

In holographic interferometry at least one of the wavefields to be compared interferometrically is stored in a hologram. The hologram may be a computer-generated hologram [13], constructed theoretically to generate a special, previously not existing wavefield. In most cases the hologram itself acts also as the beamsplitter for the superposition of the two wavefields.

### HETERODYNE HOLOGRAPHIC INTERFEROMETRY

Holographic interferometry is a powerful tool to determine the refractive index  $n(x,y,z)$  of a phase object. The relevant information is the phase difference  $\phi$  between the two optical wavefields corresponding to the two object states. These phase difference show up as intensity variation, the so-called interference fringes, in the image of the objects. However, quantitative information on the interference phase can only be obtained reliably from the maxima and minima of the interference fringes, corresponding to multiples of  $180^\circ$  or  $\pi$  in the phase. Any interpolation between the fringes is difficult and not very accurate. Heterodyne holographic interferometry is a new opto-electronic technique which overcomes this limitation and allows to determine the interference phase at any position within the fringe pattern with an accuracy of better than  $0.4^\circ$  or  $1/1000$  of a fringe [14,15].

### Heterodyne Interferometry

The basic idea of heterodyne interferometry is to introduce a small frequency shift between the optical frequencies of the two interfering light fields. This results in an intensity modulation at the beat frequency of the two light fields for any given point in the interference pattern. The optical phase difference is converted into the phase of the beat frequency [16,17].

The two light fields are then described by their complex amplitudes

$$\begin{aligned} V_1(\vec{x},t) &= a_1(\vec{x}) \exp i \left[ \omega_1 t + \phi_1(\vec{x}) \right], \\ V_2(\vec{x},t) &= a_2(\vec{x}) \exp i \left[ \omega_2 t + \phi_2(\vec{x}) \right] \end{aligned} \quad (3)$$

where  $a_{1,2}$  are the real amplitudes,  $\phi_{1,2}$  the phases, and  $\omega_{1,2}$  the optical frequencies.

A photodetector placed at the point  $P(\vec{x})$  in the superposition of these two light fields sees the time dependent intensity

$$\begin{aligned} I(\vec{x},t) &= |V_1+V_2|^2 = a_1^2(\vec{x}) + a_2^2(\vec{x}) + \\ &2a_1(\vec{x})a_2(\vec{x}) \cos \left[ (\omega_1-\omega_2)t + \phi_1(\vec{x}) - \phi_2(\vec{x}) \right] \\ &= a_1^2(\vec{x}) + a_2^2(\vec{x}) + 2a_1(\vec{x}) a_2(\vec{x}) \cos \left[ \Omega t + \phi(\vec{x}) \right]. \end{aligned} \quad (4)$$

Equation (4) shows that the interference phase  $\phi(\vec{x}) = \phi_1(\vec{x}) - \phi_2(\vec{x})$ , i.e. the optical phase difference between the two light fields, appears as the phase of the intensity modulation at the beat frequency  $\Omega = \omega_1 - \omega_2$ .

As long as  $\Omega$  is small enough to be resolved by a photodetector, this modulation can be separated by an electronic filter centered at  $\Omega$  and the phase can be measured electrically with respect to a reference signal at the same frequency. The interference phase can be measured essentially independent of the amplitude of the modulated signal and therefore also independent of the amplitudes  $a_1(\vec{x})$  and  $a_2(\vec{x})$  of the interfering light fields. The accuracy for the electronic phase measurement can easily be better than  $1^\circ$  or  $2\pi/400$ .

It is instructive to compare heterodyne interferometry and classical fringe intensity detection with respect to the accuracy of interference phase measurement. First of all it should be pointed out that in case of intensity detection the interference phase can only be reliably deduced from the position of the fringe maxima and the fringe minima, because any intermediate value of the intensity depends on both phase and average intensity, which is in general not constant across the image of the object. The heterodyne interferometry overcomes this limitation, since phase and amplitude of the interference term can be separated electronically and the fringes travel across the image so that the sensitivity and accuracy is the same at any position. This difference is visualized in Fig. 1. From the interference pattern only the positions  $x_i$  of the fringe maxima and minima  $\phi_i = m\pi$  are obtained; from heterodyne interferometry the actual interference phase  $\phi_n$  is obtained at any desired position  $x_n$ .

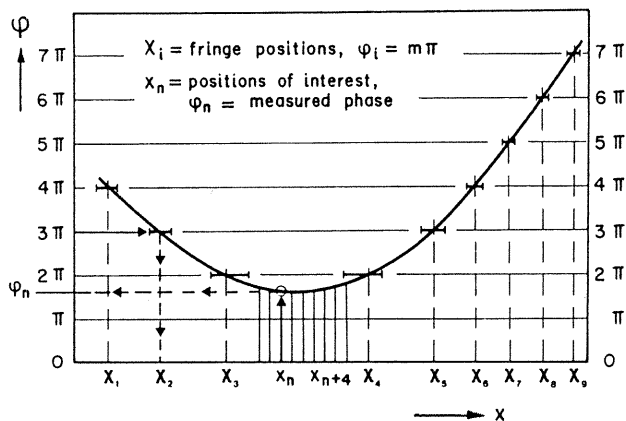


Fig. 1 Comparison of fringe counting and interference phase measurement.

#### Two-reference-beam Holographic Interferometry

Double exposure holographic interferometry probably is the most common and convenient kind of holographic interferometry. It is therefore very important to find a solution to use the described heterodyne method together with double exposure holographic interferometry. This is possible if the two wavefields are stored independently in the hologram, so that during reconstruction the different frequencies for the two interfering light fields can be introduced by using two reference waves of different frequencies. The most convenient realization is to use two different reference waves for that purpose [14,18].

Multiple-reference-beam holography has been proposed and applied by various researchers for inspecting phase objects, holographic recording of polarization, or as means for introducing flexibility into

conventional double exposed holograms. Indeed, if each image has its own reconstruction beam, one has access to each image separately, as well as to their mutual interference pattern. A double-exposure double-reference-beam holographic setup, as shown in Fig. 2, is basically a superposition of two independent holograms of the same, but slightly changed, object on the same hologram plate.

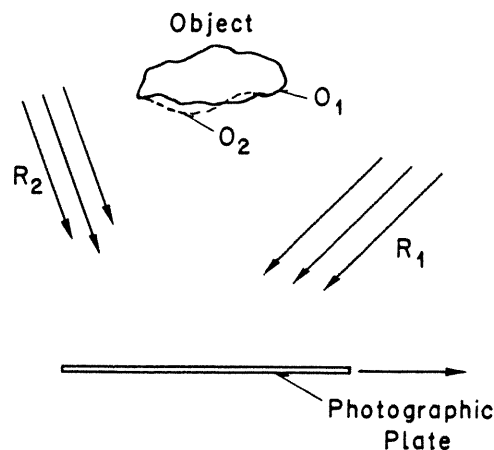


Fig. 2 Setup for recording double exposure holograms with two reference beams.  $R_1$  and  $O_1$  are used for the first exposure, while  $R_2$  and  $O_2$  are used for the second one.

The hologram  $H(\vec{x})$ , which is the interference of object and reference waves in the hologram plane  $(x,y)$ , consists in this case of

$$H(\vec{x}) = R_1 O_1^* + R_1^* O_1 + R_2 O_2^* + R_2^* O_2, \quad (5)$$

where  $R_1$  and  $R_2$  are the two different reference waves used to record  $O_1$  and  $O_2$ , respectively.

Upon illumination of the hologram with the two reference beams  $R_1$  and  $R_2$  eight terms result from the product of  $R_1+R_2$  with  $H(\vec{x})$ . Two pairs of reconstructions,  $R_1 R_1 O_1$ ,  $R_2 R_2 O_2$  and  $R_2 R_1 O_1$ ,  $R_1 R_2 O_2$ , will be in exact register giving rise to interference. The images  $R_1 R_1 O_1$ ,  $R_2 R_2 O_2$  and  $R_1 R_1 O_1$ ,  $R_1 R_2 O_2$  are the primary and conjugate self-reconstructions, respectively, of the two independent holograms with their own reference waves. In addition,  $R_2 R_1 O_1$ ,  $R_1 R_2 O_2$  and  $R_2 R_1 O_1$ ,  $R_1 R_2 O_2$  are the primary and conjugate cross-reconstructions, respectively, of the two holograms with the wrong reference waves. The locations of the reconstructed images depend on the mutual position of the reference waves and the object during recording.

Two-reference-beam holographic interferometry is expected to be sensitive to repositioning errors for the reconstruction process because misalignments of hologram and reference waves will not affect both reconstructed waves in the same manner. Angular misalignment of both hologram plate or reference waves causes mainly an additional linear phase deviation across the hologram between the reconstructed wave fields.

Experimental Setup. Two-reference-beam holographic interferometry is applied in the following way to realize the heterodyne method for double exposure holographic interferometry, as shown in Fig. 2. The first object state  $O_1$  is recorded using beam

$R_1$  as a reference. A stop is placed in beam  $R_2$  so that it does not illuminate the hologram plate. The second object state  $O_2$  is recorded in the same manner, except that beam  $R_2$  is used as a reference and beam  $R_1$  is stopped. All light fields during recording have the same optical frequency  $\omega$ . After processing the hologram is reconstructed with geometrically identical reference waves  $R_1$  and  $R_2$ , but of slightly different optical frequencies  $\omega_1$  and  $\omega_2$ , respectively. The reconstructed wave fields  $O_1$  and  $O_2$  have the same frequencies as their respective reference waves. This meets precisely the conditions necessary for heterodyne interferometry between these two reconstructed wavefields. The frequency difference  $\Omega = \omega_1 - \omega_2$  has to be small enough to be resolved by photodetectors ( $\Omega/2\pi \leq 100$  MHz). The relative frequency change  $\Omega/\omega_{1,2}$  is therefore smaller than  $2 \times 10^{-7}$  for visible light. The resulting changes in the propagation of the light waves are thus very much below any optical resolution and can be completely neglected.

### EXPERIMENTAL REALIZATION AND RESULTS

Besides the standard holographic techniques and equipment one needs for heterodyne holographic interferometry also methods to generate the desired frequency offset  $\Omega$ , to detect the modulated signals, and to measure their phase accurately. For small frequency offsets ( $\Omega \leq 2\pi \times 1$  kHz) a mechanically rotating  $\lambda/2$ -plate and subsequent polarizing elements can be used as in the early experiments of heterodyne interferometry [16]. For the sake of stability, accuracy, and measuring speed, however, larger frequency offsets ( $\Omega \approx 2\pi \times 100$  kHz) are advisable. This can be realized adequately only with either rotating radial gratings [19] or acousto-optical modulators [20]. The disadvantage of the rotating radial grating is the fixed intensity ratios between the different object and reference beams, given by the diffraction efficiency of the grating, which does not allow to optimize the light economy for both recording and reconstruction independently. Moreover, most rotating radial gratings show residual amplitude modulation due to grating imperfection which may disturb the phase measurement eventually.

For the phase measurement one needs a signal to act as a reference. Therefore at least two photodetectors are placed in the image of the object under investigation. One detector may be at a fixed position while the other scans the image, or both detectors may be movable at a fixed relative mutual separation. The latter measures rather fringe density or slope of the interference phase function than the interference phase itself. Nevertheless it has the advantage to be less sensitive to slight variations of the position of the detectors, as long as their mutual separation is kept constant, and to yield directly local derivatives. The integral interference phase can be calculated sufficiently accurate by summation of the increments.

All electronic amplifiers and filters in the signal paths should be designed carefully to avoid phase distortion which could reduce the accuracy of the phase measurements. Especially narrow band filters for noise reduction should be avoided. A bandwidth of somewhat less than half the modulation frequency  $\Omega$  is advisable to cut down the D.C. component and harmonics. The phase is measured either by a phase sensitive detector and a calibrated, variable phase shifter or more conveniently by a zero-crossing phasemeter. Both kinds of instruments are commer-

cially available with resolution down to  $0.1^\circ$  for the phase.

Besides the phase error due to amplitude noise additional phase fluctuations may occur in the signal. These are mainly caused by instabilities, e.g. in the path length if the two reference waves, and by mechanical instabilities of the position of detector and reconstructed image. The overall accuracy of phase measurement, including these phase fluctuations, has recently been investigated experimentally. The experimental verification of heterodyne holographic interferometry was made. The results indicated an accuracy of interference phase of  $\delta\phi = 0.3^\circ$ , which corresponds to less than 1/1000 of a fringe.

A somewhat more advanced setup for heterodyne holographic interferometry is shown in Fig. 3. In this case the frequency shift of  $\Omega = 2\pi \times 100$  kHz is realized by two commercially available acousto-optical modulators  $M_1$  and  $M_2$  in cascade to give opposite frequency shift. During recording both modulators are driven with 40 MHz, so that the net shift is zero. During reconstruction one modulator is driven with 40 MHz and the other one with 40.1 MHz, so that the net shift is the desired 100 kHz. An array of detectors is used to scan the image. The detectors  $D_1, D_2$  and  $D_3$  in Fig. 3, get their light from the detection points in the image plane by optical fibre-bundles. The signal amplitudes are kept approximately constant independent of the intensity across the image by control of the detector voltage. The phase difference  $\psi$  is measured with a zero-crossing phasemeter, which interpolate the phase angle to  $0.1^\circ$  and count also the multiples of  $360^\circ$ , which corresponds to the fringe number.

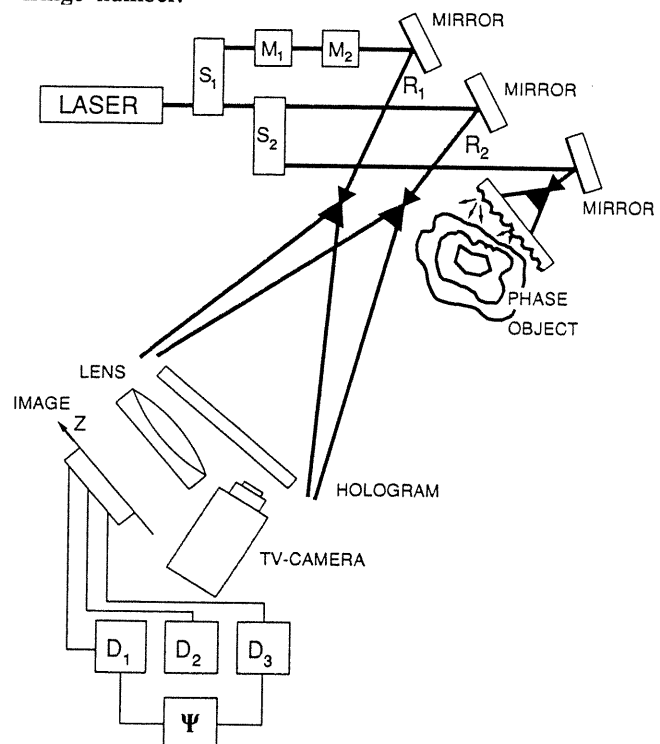


Fig. 3 Experimental setup for heat transfer measurement with heterodyne holographic interferometry. The two reference waves  $R_1$  and  $R_2$  are generated by polarizing beam splitters. Frequency shift is realized by two acousto-optical modulators  $M_1$  and  $M_2$ .  $D_1$ ,  $D_2$  and  $D_3$  are photodetectors in the image plane of the object.

Heat Transfer Measurements

In order to demonstrate the use of heterodyne holographic interferometry for precise heat transfer measurement, the thermal boundary layer on a heated horizontal flat test plate in air is investigated.

The experimental arrangement for heat transfer measurements is shown schematically in Fig 3. The beam of a 500 mW single frequency Ar-laser ( $\lambda = 514$  nm) is split in three beams by two polarizing beam-splitters. The frequency shift is realized by two acoustooptical modulators  $M_1$  and  $M_2$  in one of the two reference beams.

The test chamber consists of a transparent cylindrical tube (200 mm in diameter), a heat source, and a horizontal test plate, as shown in Fig. 4. The test plate, which is held at the top by isolating material attached to a laboratory stand, is aligned horizontally. The plate test section consists of a 8 cm by 5 cm rectangular, 5 mm thick titanium plate. The titanium plate is heated by a burner 25 cm in distance. Thermocouples are attached ( $T_1, T_2$  and  $T_3$ ) to indicate the temperature and the surface temperature and the spatial variation of the temperature.

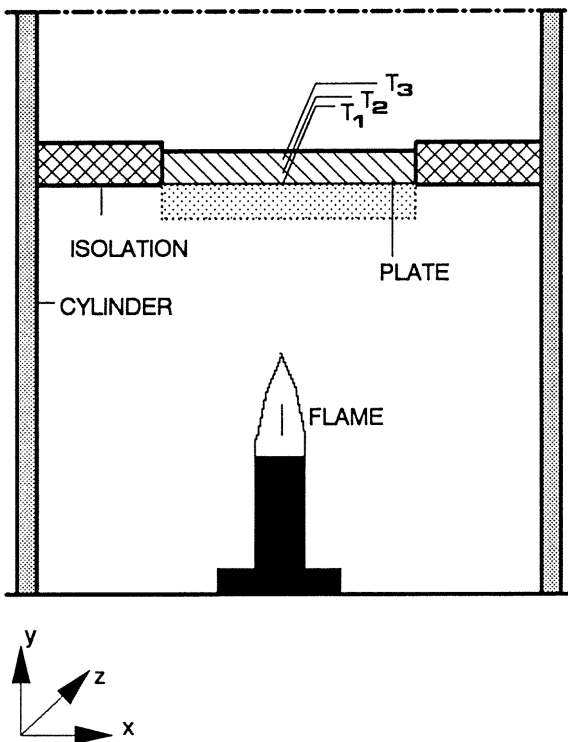


Fig. 4 Test object: transparent cylindrical tube with heat source and horizontal test plate.

The double exposure holograms have been recorded at a two different temperatures, measured with the help of the thermocouples. The holographic reconstruction is shown in Fig. 5.

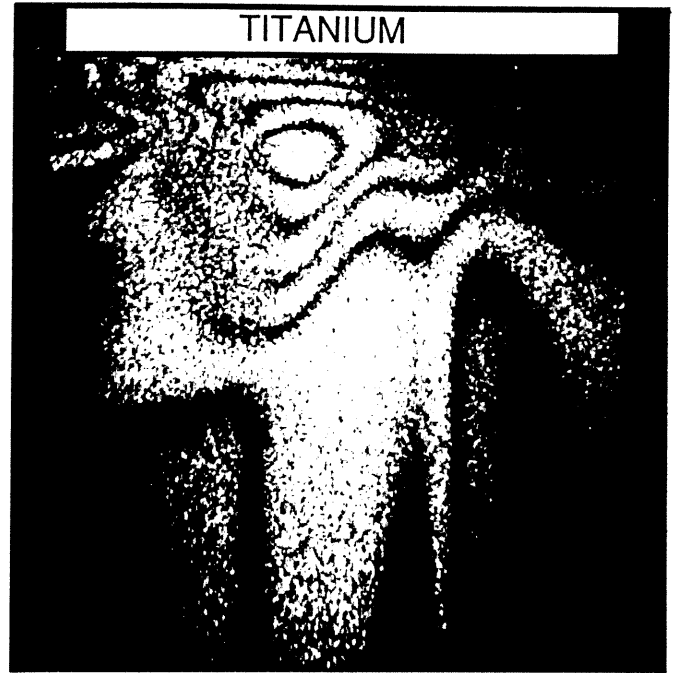


Fig.5 Holographic interferogram of the temperature field in air near a titanium plate.

A computer program has been developed for the numerical evaluation of the measured data to determine the spatial distribution of temperature. Typical results for the temperature distribution horizontal and vertical to the test plate are displayed in Fig. 6 and 7.

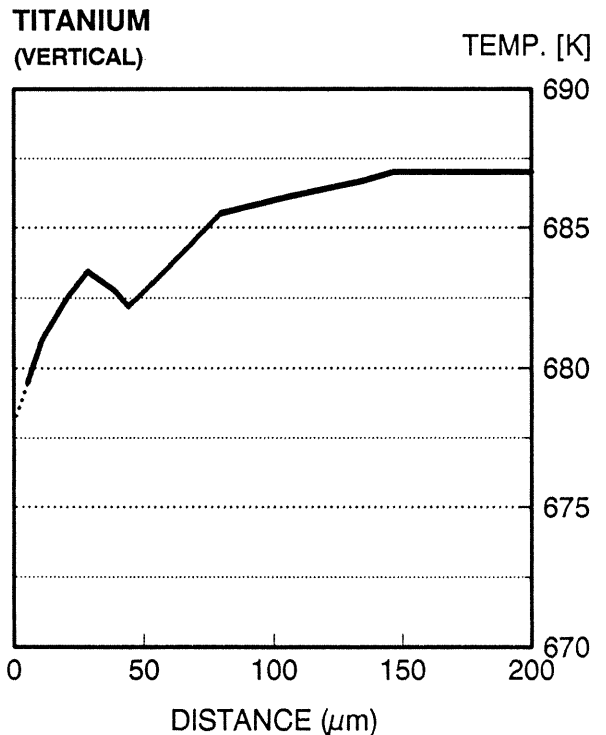


Fig.6 Vertical temperature distribution in the center of the titanium test section.

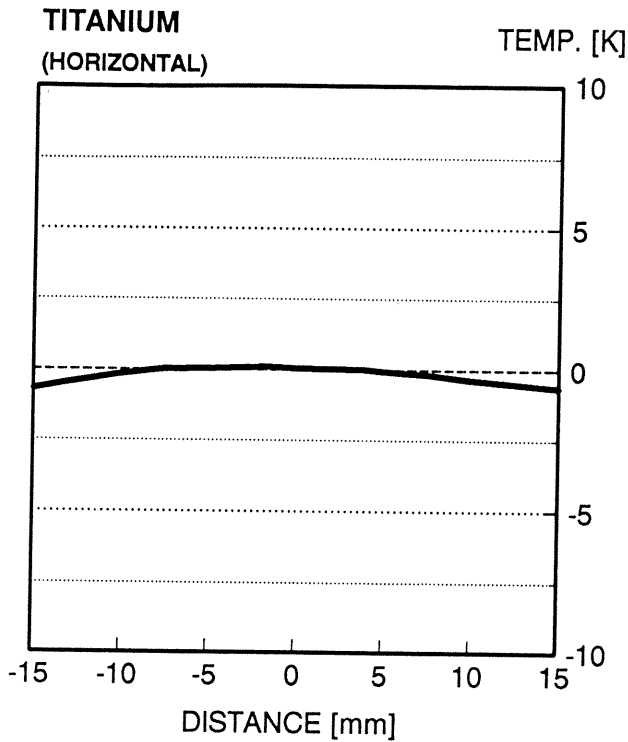


Fig.7 Horizontal temperature distribution 15  $\mu\text{m}$  below the titanium plate.

The final output of the developed computer program, which is shown in Fig. 8, allows to present the 3D temperature distribution in the boundary layer of the titanium plate.

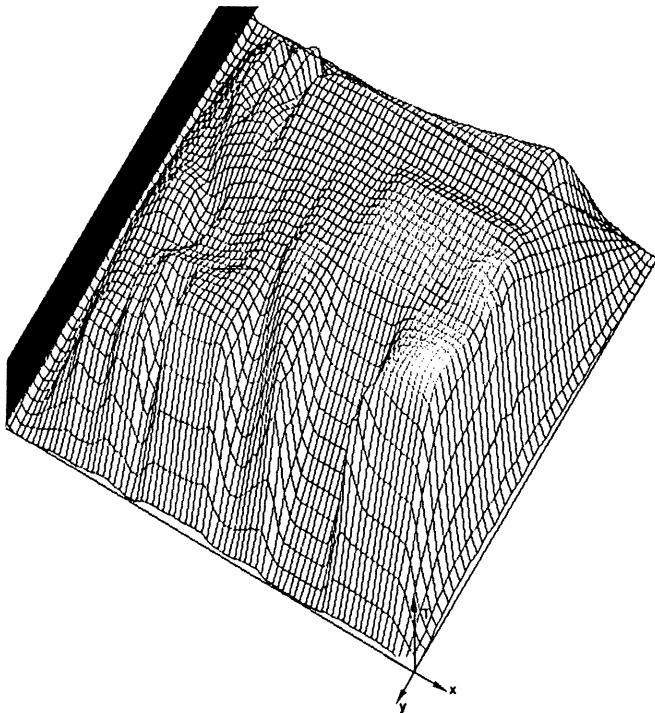


Fig.8 3D temperature distribution in the boundary layer of the titanium plate.

A method is being developed for coupling conduction in a solid with natural convection in a surrounding fluid. The problem investigated so far is that of steady, constant property, two-dimensional, natural convection from a horizontal flat plate of finite thickness with an arbitrary heating distribution on its surface. Preliminary results using this method show that it is possible to predict the variation of temperature profiles in the boundary layer as a function of the heating distribution and the physical properties of the plate and fluid. The equations for conduction in the plate and convection in the boundary layer are written in finite difference form, coupled through the common heat flux at the plate-fluid interface, and solved numerically by an iterative technique. A preliminary numerical result is shown in Fig. 9 and compared with heterodyne holographic interferometry data.

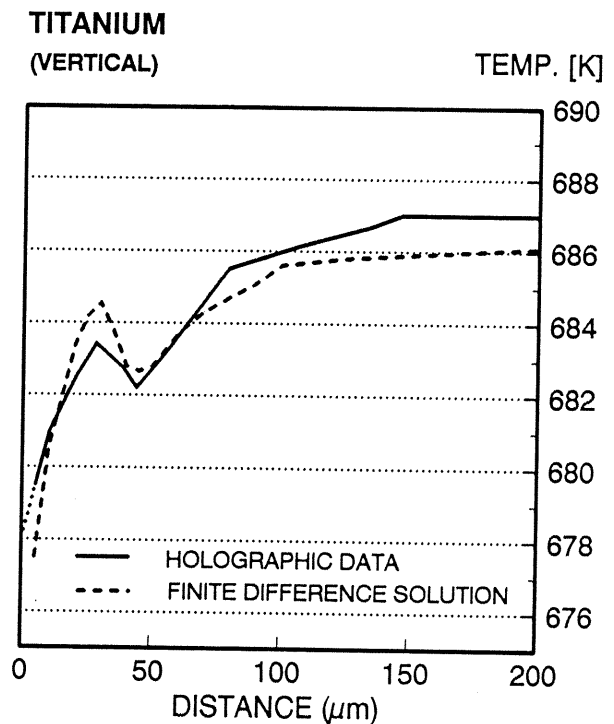


Fig.9 Vertical temperature distribution in the center of the titanium test section. Experimental results are given by the solid line and preliminar calculated results by the broken line.

CONCLUSIONS

It has been shown and experimentally verified that heterodyne holography has the following outstanding properties:

- fringe interpolation to better than  $10^{-3}$  of a fringe ( $\delta\phi = \pm 0.3^\circ$ ),
- measurement with the same accuracy at any desired position in the image, therefore high spatial resolution,
- independent of brightness variations across the image,

- inherently direction sensitive, i.e. increase and decrease of interference phase can be distinguished,
- computer readable output both for position and phase easily obtained (allows also on-line data processing),
- inherently less sensitive to speckle noise than fringe intensity measurement.

For these reasons heterodyne holographic interferometry is considered to be a powerful technique to collect data for quantitative reconstruction of three-dimensional refractive index distributions in transparent objects allowing temperature profile measurements in the boundary layer.

#### ACKNOWLEDGMENTS

The authors would like to express their appreciation to Prof. Dr. M.K. Eberle for his support of this investigation. We wish to thank E. Sannemann and P. Eberli for their help with the experiment. They are also indebted to P. Obrecht for the preparation of the thermocouple amplifier electronics.

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