Case History	Measurement of Bending-Torsion Coupled Resonance of Rotor	Rotating machinery
Resonance		

Object Machine

Rotor having two circular disks at both ends (Fig.1)

Observed Phenomena

Let the torsional natural frequency be f_θ and the bending natural frequency be f_b. Bending excitation of the rotor at f_b when the rotational speed of the rotor f_r was equal to $|f_\theta \pm f_b|$ may generate not only bending vibration but torsional vibration due to coupled resonance. Thus, an experiment was conducted to verify this phenomenon.

The specification of the rotor for this experiment is given in Table 1. This rotor was supported by a ball bearing and an active magnetic bearing, and was provided with one circular plate on each end.

When the rotor was bending-excited at f_{b1} =18.0[Hz] for $f_r = |f_{\theta1} \pm f_{b1}| = 37.3/73.3$[rps] , beat noise that was apparently caused by torsional vibration was heard from the rotor, but the current apparatus was unable to measure the torsional vibration.

Analysis and Data Processing

The natural frequencies of the bending and torsional vibrations f_{nr} when viewed from the rotating field vary as illustrated in Fig.2 (in case $f_\theta > f_b$). That is, the bending natural frequency f_b decreases according to the increase in f_r, while the torsional natural frequency f_θ remains constant independent of f_r.

As shown in Fig.2, at the intersection points P ($f_r = f_\theta - f_b$) and Q ($f_r = f_\theta + f_b$), the bending vibration and the torsional vibration are mutually coupled.

Countermeasures and Results

Verification of the bending-torsion coupled vibration required the measurement of torsional vibration. Thus, as shown in Fig.3, a slit disk was attached at one end of the rotor, so as to pick up pulse signals through a photo-interrupter. As the rotor rotated while being accompanied by torsional vibration, the pulse signals were compressional waves (Fig.4 top).

Let the rotor torsional frequency be f_t [Hz] and the torsional angle be Δ [rad], thus the rotor rotational angle being $\phi = 2\pi f_r t + \Delta\cos(2\pi f_t t)$, while the rotational angular velocity being $\dot{\phi} = 2\pi f_r - \Delta 2\pi f_t \sin(2\pi f_t t)$. Where the number of teeth of the slit disk is N, the angular velocity of the pulse signal will be N times the rotational angular velocity, so that it becomes $N\dot{\phi} = N2\pi f_r - N\Delta 2\pi f_t \sin(2\pi f_t t)$. The alternating current component $N\Delta 2\pi f_t \sin(2\pi f_t t)$ of the demodulated wave extracted by F/V conversion [(1)] of the pulse signal (Fig.4 middle) is the torsional vibration, whose amplitude is proportional to N. The sensitivity is enhanced for increasing N, so that the slit disk was designed for N = 360.

Fourier transformation of the above alternating current component of the demodulation wave allows the magnitude and frequency of the torsional vibration to be determined (Fig.4 bottom).

Using the above torsion measuring instrument, it was confirmed that, when the rotor was bending-excited at fb1 =18.0[Hz] for $f_r = |f_{\theta1} \pm f_{b1}| = 37.3/73.3$[rps] , not only bending vibration but torsional vibration occurred (Fig.5).

Lesson

In case $f_r = |f_\theta \pm f_b|$ is present within the rated operating speed of the rotor, it is necessary to pay attention to the rotor bending-torsion coupled resonance.

References

(1) Instruction Manual on Delta F/V Transducer (2nd edition) (UNICO JAPAN Publishing)

Keyword

Bending-torsion coupling, active magnetic bearing

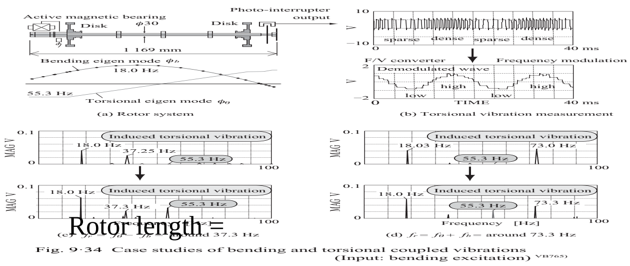


Fig.1: Experimental rotor

Table 1: Specification of the rotor for this experiment

Total weight	33.4 kg
Shaft length	1.17 m
The 1st bending natural frequency : f_{b1}	18.0 Hz
The 1st torsional natural frequency : $f_{\theta 1}$	55.3 Hz

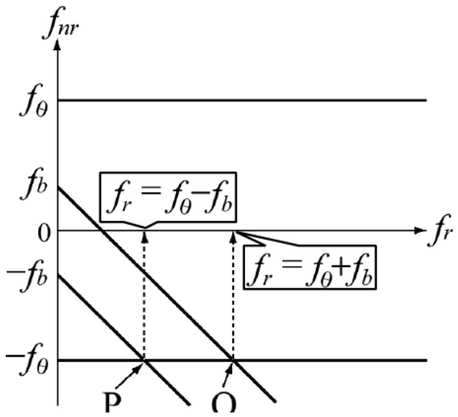


Fig.2: Transition of natural frequencies in the rotating field

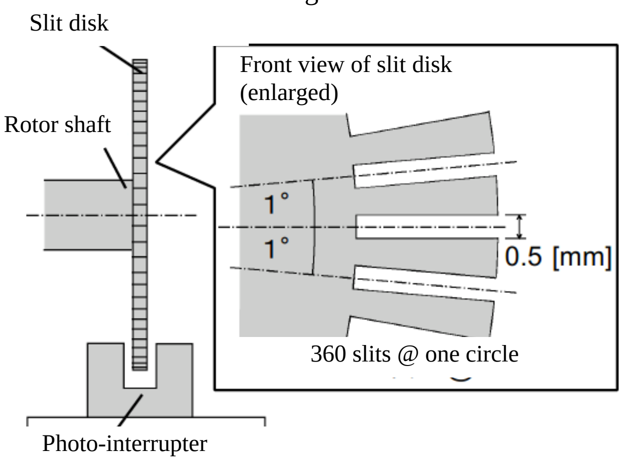


Fig.3: Slit disk and photo-interrupter

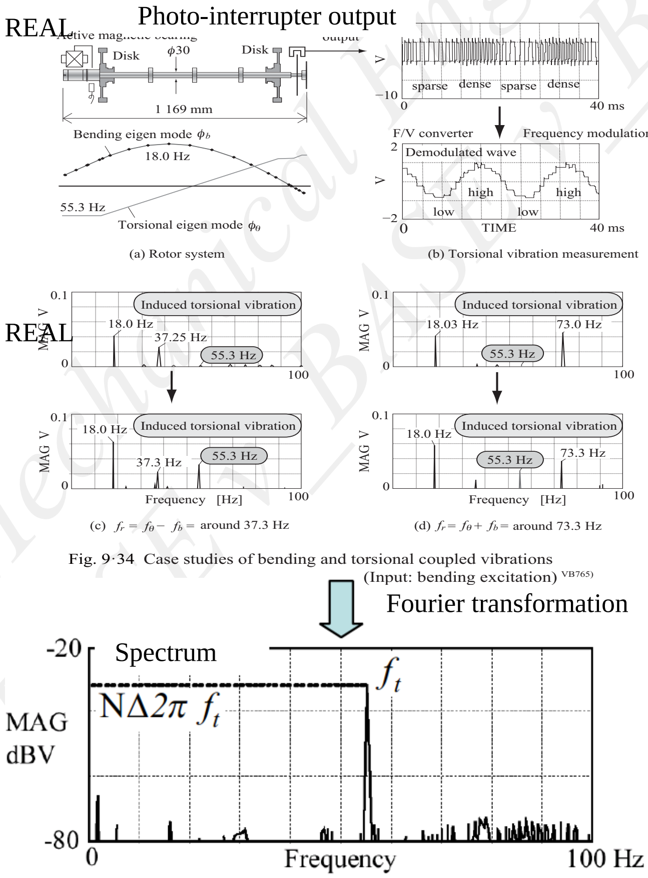
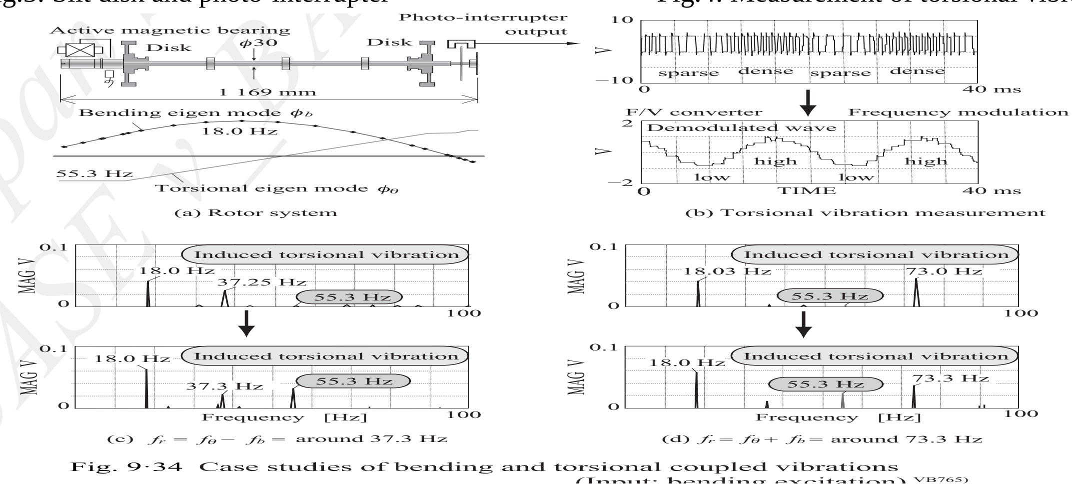


Fig.4: Measurement of torsional vibration



(1) $f_r = |f_{\theta 1} - f_{b1}|$ =37.3[rps]

(2) $f_r = |f_{\theta 1} + f_{b1}|$ =73.3[rps]

Fig.5: Generation of torsional vibration due to bending excitation