Vibration Case	Circle to Ellipsoid in Polar Plot of Asymmetric Stiffness Rotor	Rotating machinery
Self-excited		

Object Machine

A high speed rotor having a plurality of flexible couplings

Observed Phenomena

A modal circle in Nyquist diagram measured when passing the critical speed during acceleration was characterized by an extreme ellipsoid (Fig.1). In some cases, this resulted in divergence without being able to pass the critical speed.

Cause Estimation

The ovalization phenomenon was characterized in that the ellipsoid tended to be small for a small unbalance, while the ellipticity and the direction of major axis and minor axis remained unchanged. As this was a phenomenon related to stability, instability of the asymmetric rotor was estimated to be the cause. The only cause for asymmetricity that came to mind was the asymmetricity of the bending stiffness of flexible couplings.

Analysis and Data Processing

The equation of motion of an asymmetric rotating shaft is similar to the Mathieu equation having stiffness variations of twice the rotational speed Ω, where circular whirl with the rotational speed Ω is assumed and the complex variable $Z = X + iY$ is introduced. Considering an equivalent damping (damping ratio ζ), the following equation is obtained:

$$\ddot{Z} + 2\zeta\,\omega_n\dot{Z} + \omega_n{}^2\left(Z - \mu\, e^{i2\Omega t}\,\overline{Z}\right) = \varepsilon\,\Omega^2 e^{i(\Omega t+\beta)} \quad \text{where} \quad \mu = \frac{\omega_y{}^2 - \omega_x{}^2}{2\,\omega_n{}^2}, \quad \omega_n{}^2 = \frac{\omega_y{}^2 + \omega_x{}^2}{2} \qquad (1)$$

$\overline{Z} = X - iY$, ω_n is average natural frequency, thus mode asymmetricity μ can be obtained from the natural frequencies in the x (flexible) and y (stiff) directions.

By putting $Z = (x + iy)e^{(i\Omega+\lambda)t}$, the stability condition can be derived from the following the Hurwitz stability criterion of the characteristic equation concerning λ:

$$\left(1 - (\Omega/\omega_n)^2\right)^2 + 4\zeta^2(\Omega/\omega_n)^2 - \mu^2 > 0 \qquad (2)$$

The left hand-side of the above equation = 0 (stability & instability limit) is a quadratic equation on $(\Omega/\omega_n)^2$, which can thus be solved as follows:

$$(\Omega/\omega_n)^2 = 1 - 2\zeta^2 \pm \sqrt{\mu^2 - 4\zeta^2\left(1 - \zeta^2\right)} \approx 1 - 2\zeta^2 \pm \sqrt{\mu^2 - 4\zeta^2} \qquad (3)$$

The stability condition independent of Ω is $\mu^2 - 4\zeta^2 < 0$, so that the stability condition is given by effective asymmetric ratio $\kappa = \mu/(2\varsigma) < 1$.

The reason for ovalization is that the restoring force does not tend toward the original point, and that the effective damping force fluctuates (refer to Fig.2). In this case, κ and μ were as shown in Figs.3 and 4. It is known that the ellipticity *A* and κ are related as follows[(2)]:

$$\boldsymbol{A} = \sqrt{\frac{1+\kappa}{1-\kappa}}, \text{ thus } \kappa = \frac{\boldsymbol{A}^2 - 1}{\boldsymbol{A}^2 + 1} \qquad (4)$$

Countermeasures and Results

The countermeasures consist of reducing the modal asymmetric ratio μ and of increasing the damping ratio ζ. In this case, the anisotropy of bending stiffness of the flexible rotor was measured and controlled under a small value.

Lesson

In this phenomenon, the modal asymmetric ratio was μ = 0.01. Low damping ratio causes various problematic phenomena.

References

(1) R Gasch, H Pfützner, “Rotordynamik — eine Einführung”, Springer-Verlag (1975)
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(3) Iwatsubo, et al. "Study on Passing Critical Speed of Asymmetric Shaft with Finite Drive Force", *Transactions of the JSME*, Vol.40, Series 335, (1974), 1908 (in Japanese)

Keyword

Anisotropic stiffness rotor, parametric excitation, Mathieu equation, Hill equation, modal circle

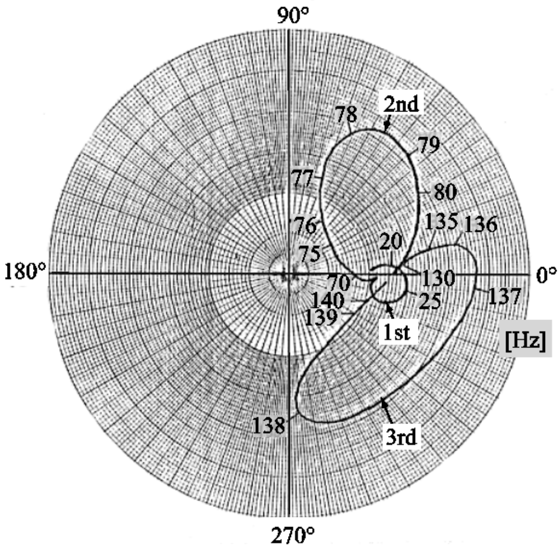

Fig.1 Example of ovalization of modal circle (numerical figures: rotational speed in Hz)

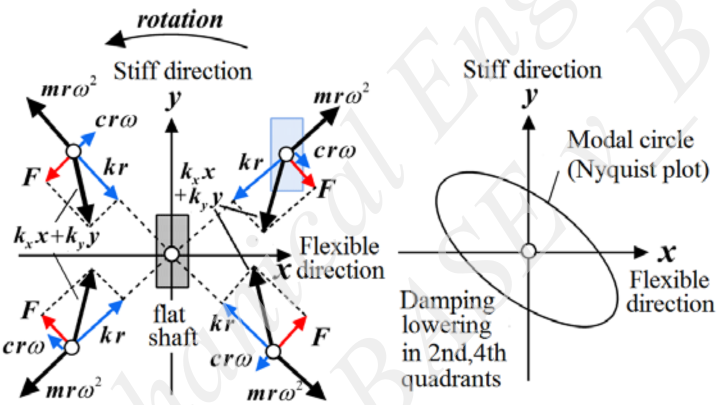

Fig.2 Figure explaining why a modal circle changes to an ellipsoid

(A flat shaft generates a mismatch between the directions of shaft deflection and of restoring force, thus causing a force F in the whirling direction. When taking the stiff axis and flexible axis directions in y and x, F acts for additive damping in the 1st and 3rd quadrants, while for cancelling the damping in the 2nd and 4th quadrants.)

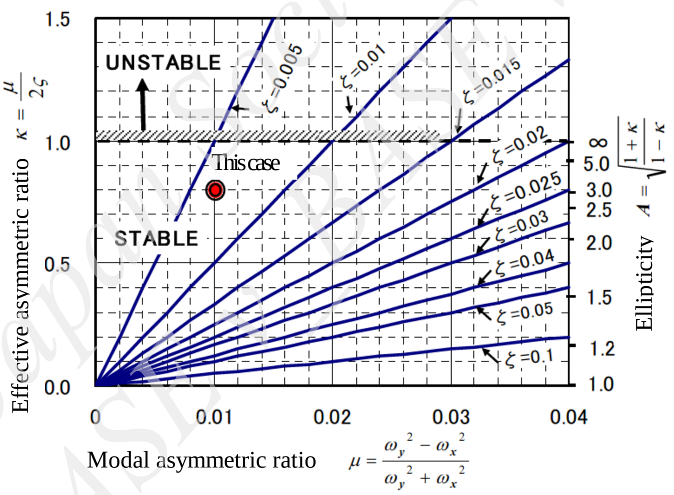

Fig.3 Relationship among modal asymmetric ratio μ, effective asymmetric ratio κ and ellipticity A, for damping ratio ζ as a parameter

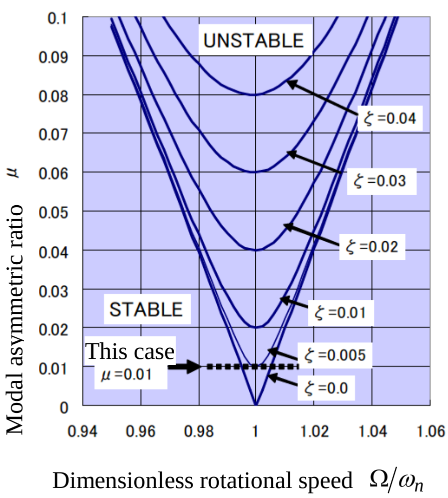

Fig.4 Stability diagram of parametric vibration for damping ratio ζ as a parameter