Case History	Vibration due to Asymmetric Stiffness of Rotating Thin Disk Fan	Rotating machinery (compressor)
Nonlinear		

Object Machine

Multi-blade fan (Fig.5) [2]

Observed Phenomena

At a certain rotational speed, the fan experienced excessive vibrations featuring a frequency twice the rotational speed measured by a displacement sensor (inertial coordinate system). Stress measured on the core plate of the fan (rotational coordinate system) proved that the vibrations had 1X frequency synchronized with the rotational speed (Fig.6).

Cause Presumed

As shown in Fig.1, a keyway causes a vibration problem due to anisotropy of shaft stiffness. The natural frequencies of the shaft in X and Y directions differ slightly. As a result, when the rotational speed reaches 1/2 of its natural frequency, the rotor experiences resonance vibration at a frequency twice the rotational speed, which is famous as a secondary critical speed due to gravity (Fig.2).
It is believed that a resonance problem similar to this phenomenon occurs on rotating structures (impeller blades, disks) such as fans. This similarity is regarded as follows.

Element	Anisotropy	Natural frequency	Acting force
Rotational shaft	Shaft deflection stiffness	Vibration of shaft deflection mode	Gravity
Fan (blade)	Tilting stiffness of thin disk, called moment spring	Vibration of nodal diameter k=1, i.e., tilting mode of blade and disk	Constant moment caused by wind pressure difference on the suction side

Analysis and Data Processing

The equation of motion of an anisotropy shafting is given on the inertial coordinate system as:

Rotational speed

(★ 1)

$$\mathbf{M\ddot{Z} + i\Omega Cg\dot{Z} + KZ + C\dot{Z} + \Delta Ke^{2i\Omega t}\bar{Z} = Mg}$$

Flexural vibration	→	Mass	Gyro	Shaft stiffness	Damping	Anisotropic stiffness	Gravity
Tilt vibration	→			Tilting stiffness			Moment

Note that (★ 1) indicates the spring force changes twice per one rotation with small magnitude.
Let deformation be Zg that is subject to a constant gravity force, the anisotropic stiffness generates a compelling force of $\Delta Ke^{2i\Omega t}\bar{Z}g$, so that resonance occurs under the following conditions, i.e., the conditions given in Fig.3 and Fig.4:

(1) On inertial coordinate system (measured by displacement sensor):
forward natural frequency of the rotor system = rotational speed × 2

(2) On rotational coordinate system (measured by train gauge):
forward natural frequency of the rotor system = rotational speeds

Figure 7 shows the result of an FFT analysis of stresses measured by a strain gauge obtained when hitting the fan during rotation, while Figure 8 summarizes the result. Certainly, the intersection point in Fig.8 corresponds to the resonance condition, which agrees with the rotational speed for the resonance peak amplitude of the actual machine shown in Fig.6.

Countermeasures and Results

If a keyway is provided on the shaft, a dummy keyway seems to be cut so as to ensure symmetry with respect to the shaft center.
In case of a rotating disk (referring to a core plate of blade), anisotropy occurs in terms of titing stiffness due to circumferential unevenness of plate thickness. Therefore, it is a reasonable decision to eliminate the anisotropy by precise machining of the disk plate thickness, but this is not preferred because of mounting manufacturing costs.
Consequently, a measure was taken to enhance the natural frequency by applying a stiffening rib on the rear side of the disk core plate by welding. If we are during the design stage and before the manufacturing stage, it is relatively easy to raise the natural frequency of one nodal diameter in order to avoid this resonance condition by properly selecting the core plate thickness and diameter.

Lesson learned

Rotating bodies should always be manufactured with precision in terms of axial symmetry.
Since the natural frequency of one nodal diameter of a disk at standstill is divided into two natural frequencies due to gyro effect, i.e., forward one tending to increase and backward one tending to decrease according to the increase of the rotation, it is necessary to deliberately perform the predictive calculation of resonance conditions.
A plan should be made to allow hitting during the rest and the rotation, so as to measure the natural frequency of rotors. It would be much better to carry out a hitting test by changing the rotational speed.

References

[1] Gash (translated by Miwa). 1978-9. *Dynamics of Rotating Bodies*: Morikita Publishing Co. 129p.

[2] Hagiwara et al. 1981-11. Resonance Phenomenon in Forward & Backward Whirl Mode of Impellers. *Transactions of the JSME* 47(423): 1,457

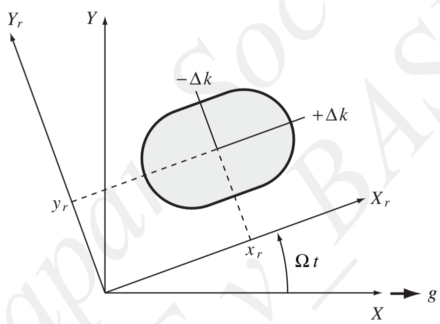


Fig.1: Asymmetric axis of rotation

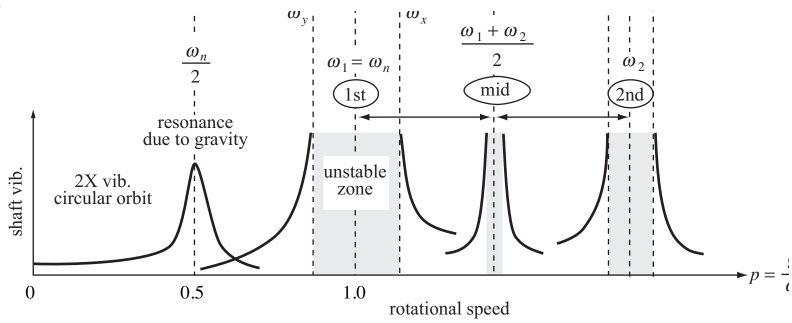


Fig.2: General behavior (qualitative) of non-true circle axis placed horizontally [1]

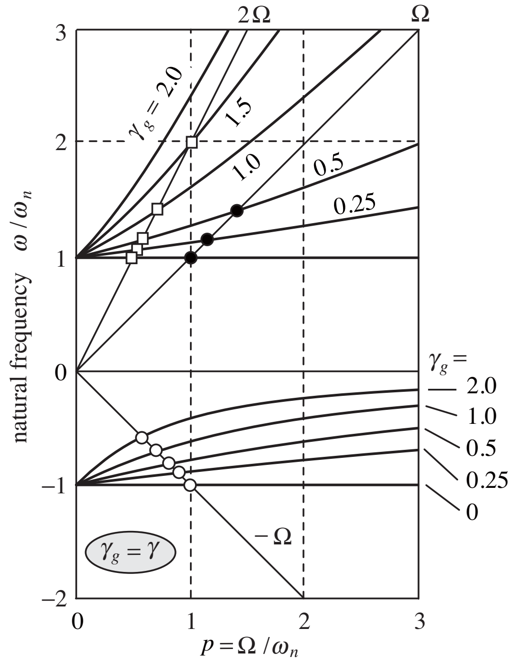


Fig.3: Natural frequency on the coordinate system at rest

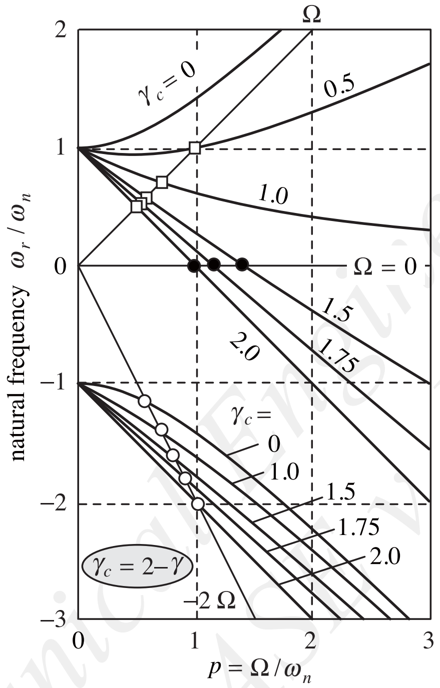
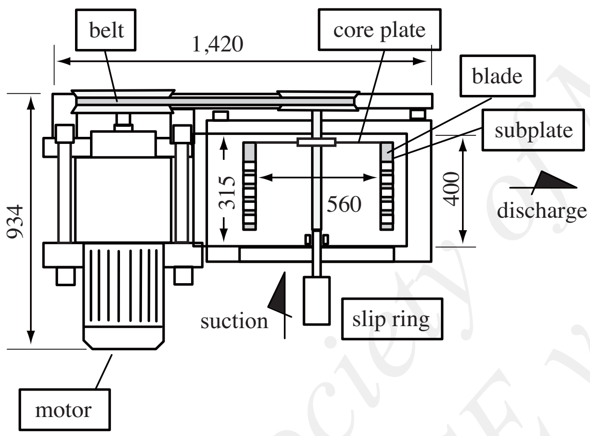


Fig.4: Natural frequency on the rotational coordinate



core plate thickness = 3.2 mm   blade thickness = 2.3 mm

subplate number = 56   blade number = 56

Experimental apparatus of multi-blade fan

Fig.5-1: Multi-blade fan

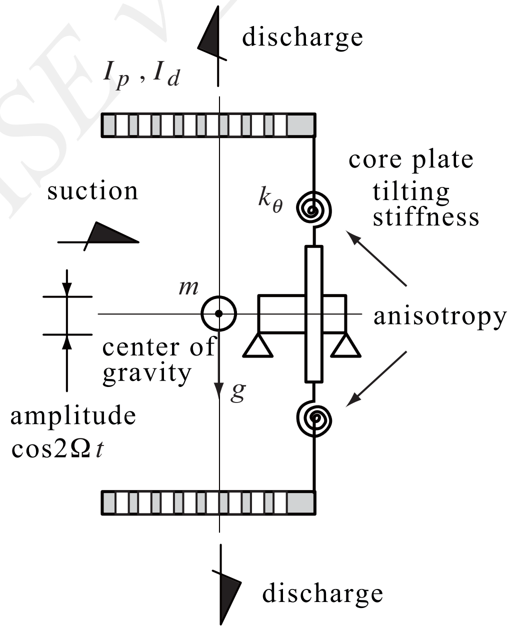
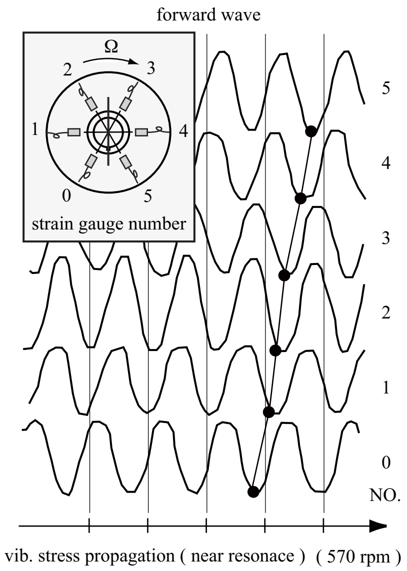
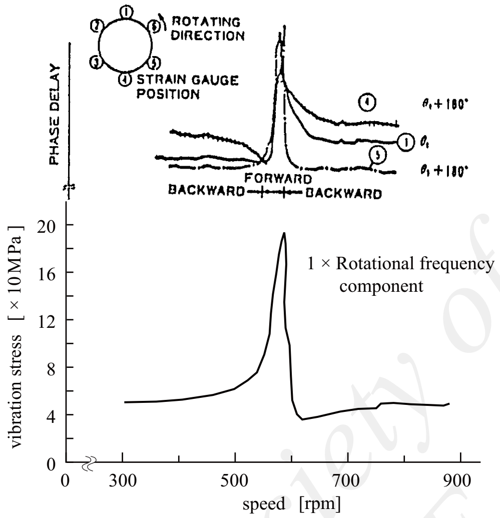


Fig.5-2: Fan model



Waveform at the time of resonance



Resonance response of multi-blade fan due to gravity

Fig.6: Response of strain gage

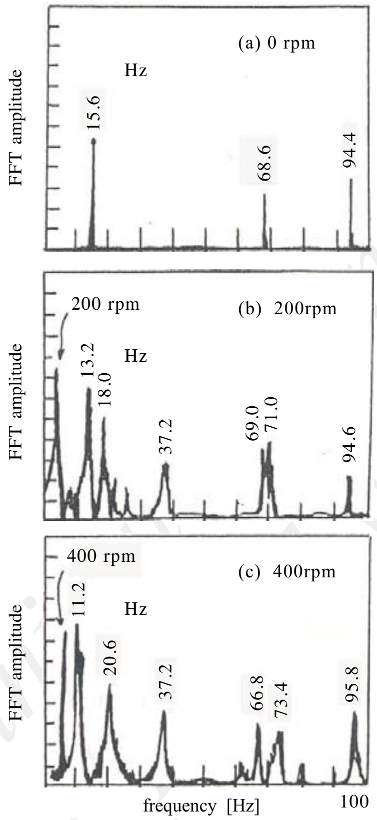


Fig.7: Spectrum due to impulse excitation (numbers in the figure indicate frequencies in Hz)

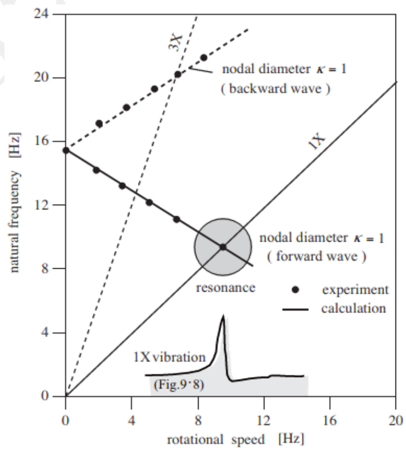


Fig.8: Comparison of experimental values of natural frequencies of one nodal diameter with calculated values