AN ITERATION METHOD FOR SINGULAR FIELDS AROUND AN INTERFACE EDGE OF ELASTIC/POWER-LAW HARDENING MATERIALS JOINT

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1. INTRODUCTION:

The objective of present study is to present solution to determine the stress and displacement fields around an interface edge of material joint formed by quarter planes in which materials behaves as an elastic and a power-law hardening material. On the interface, a separable form solutions of stress, $\sigma_{ij} \propto r^{\lambda-1} f\left(\theta\right)$, gives stress continuity but, due to the dissimilarity of power of r the displacement does not become continuous. Recently, many researchers have investigated the elastic-plastic stress singularity of an interface crack between two bonded power law hardening materials.In order to satisfy both the continuous conditions of the traction and the displacement at the interface edge, Duva[1], Rahman[2] and Reedy[3] modelized the elastic/power-law hardening plastic materials joint as a power law hardening plastic material on a rigid substrate.They conducted the asymptotic analysis similar to the nonlinear crack problem developed by Hutchinson[4].In other study, Liton et al.[5] studied experimentally and numerically the characteristics of singular fields around an interface edge between ceramic/metal joint.

In the present study, governing differential equation obtained from compatibility condition is solved theoretically to satisfy the continuity of displacement and the balance of force on the interface between an elastic material and a power-law hardening materials joint. In order to satisfy both the continuity condition of stresses and displacements on the interface, the successive functions as eigenfunction expansion method has been used.

2. BOUNDARY CONDITIONS AND METHOD OF SOLUTION:

The boundary condition can be expressed as follows in the polar coordinate system located on the interface edge for elastic/power-law hardening materials joint,

$$\left(\sigma^{I}_{\theta\theta}\right)_{\theta=\frac{\pi}{2}} = \left(\sigma^{II}_{\theta\theta}\right)_{\theta=-\frac{\pi}{2}} = 0, \qquad \left(u^{I}_{r}\right)_{\theta=0} = \left(u^{II}_{r}\right)_{\theta=0}, \qquad \left(\sigma^{I}_{\theta\theta}\right)_{\theta=0} = \left(\sigma^{II}_{\theta\theta}\right)_{\theta=0},$$

$$\left(\sigma^{I}_{r\theta}\right)_{\theta=\frac{\pi}{2}} = \left(\sigma^{II}_{r\theta}\right)_{\theta=-\frac{\pi}{2}} = 0, \qquad \left(u^{I}_{\theta}\right)_{\theta=0} = \left(u^{II}_{\theta}\right)_{\theta=0}, \qquad \left(\sigma^{I}_{r\theta}\right)_{\theta=0} = \left(\sigma^{II}_{r\theta}\right)_{\theta=0}.$$

The stresses and displacements of power-law hardening material are reffered to with a superscript “I” while those of the elastic material, with a superscript “II”. In the zero-th order approximation, the power-law hardening material of the elastic/power-law hardening materials joint is considered as a power-law hardening material on a rigid substrate. The stress fields in the elastic material can be described by the fields of an elastic wedge which is subjected to distributed tractions along the one edge. The magnitudes of the tractions are the same as the stress distributions along the rigid/power-law interface. In the first order approximation, the power-law hardening material having the initial fields are subjected to a forced displacement which is the field on the edge of elastic material of the zero-th order approximation was considered. The increase of stress fields in the elastic material can be described by the fields of an elastic wedge which is subjected to distributed tractions along the one edge. The magnitudes of the traction are the same as the stress distributions along the power-law material wedge.

As the joint material model of elastic/power-law hardening materials joint, joint of elastic and elastic-plastic plate was considered for the simulation in FEM, where uniform tension load was applied as the external load on the one edge of the joint plate. Stress-strain relations used in the elastic-plastic side is $\varepsilon_{ij}^{I} = {}^{3}\!/_{2}\,\alpha\sigma_e^{n-1}s_{ij}^{I}$ and elastic side is $\varepsilon_{ij}^{II} = {}^{E^I}\!/_{E^{II}}\left\{\left(1+\nu^{II}\right)\sigma_{ij}^{II} - \nu^{II}\sigma_{kk}^{II}\delta_{ij}^{II}\right\}$. where i , j and k are used for subscript indicates r,θ. E is Young’s modulus, ν is the Poisson’s ratio, δ_{ij} is the two dimensional Kronecker delta symbol, α and n are hardening coefficient and hardening exponent, respectively. σ_e is the effective stress and s_{ij} is the stress deviator. Stress and strain quantities are normalized by yield stress or corresponding yield strain. An asymptotic expansion of the Airy stress function in a separable form is assumed $\phi^k(i) = \sum_i A_i r^{\lambda_i+1}\tilde{\phi}_i^k$, $i = 0,1,2,...;\ as\ \ r \to 0$, where $\lambda_0 < \lambda_1 ....$ and $k = I$ for power-law hardening material and $k = II$ for elastic material. A_0 and A_1 are the stress intensity factor of zero-th and first order approximation, respectively. In the higher order approximation $(i \geq 1)$, nonlinear effective stress term σ_e^{n-1} was expanded by Taylor series expansion method and the first two terms were considered for further calculations.

Before expansion this term, $\sigma_e^{n-1} = \left[\frac{3}{8}\left(r^{2(\lambda_0-1)}f_0 + 2A_1 r^{(\lambda_0+\lambda_1-2)}f_1 + A_1^2 r^{2(\lambda_1-1)}f_2\right)\right]^{\frac{n-1}{2}}$

where, $f_0 = \left(f_{0rr}^2 + f_{0\theta\theta}^2 + 8f_{0r\theta}^2\right)$, $f_1 = \left(f_{0rr}f_{1rr} + f_{0\theta\theta}f_{1\theta\theta} + 8f_{0r\theta}f_{1r\theta}\right)$, $f_2 = \left(f_{1rr}^2 + f_{1\theta\theta}^2 + 8f_{1r\theta}^2\right)$

$f_{0rr} = A_0\left\{\tilde{\phi}_0^I\left(\lambda_0+1\right)\left(1-\lambda_0\right)+\left(\tilde{\phi}_0^I\right)''\right\}$, $f_{1rr} = \left\{\tilde{\phi}_1^I\left(\lambda_1+1\right)\left(1-\lambda_1\right)+\left(\tilde{\phi}_1^I\right)''\right\}$, $f_{0\theta\theta} = -\left\{\tilde{\phi}_0^I\left(\lambda_0+1\right)\left(1-\lambda_0\right)+\left(\tilde{\phi}_0^I\right)''\right\}$

$f_{1\theta\theta} = -\left\{\tilde{\phi}_1^I\left(\lambda_1+1\right)\left(1-\lambda_1\right)+\left(\tilde{\phi}_1^I\right)''\right\}$, $f_{0r\theta} = \left\{-A_0\left(\tilde{\phi}_0^I\right)'\lambda_0\right\}$, $f_{1r\theta} = \left\{-\left(\tilde{\phi}_1^I\right)'\lambda_1\right\}$

After expansion and neglecting the higher order term of A_1 the equation becomes,

$$\sigma_e^{n-1} \approx \left[\frac{3}{8}r^{2(\lambda_0-1)}f_0\right]^{\frac{n-1}{2}} + \frac{n-1}{2}\left[\frac{3}{8}r^{2(\lambda_0-1)}f_0\right]^{\frac{n-3}{2}} \times \frac{3}{4}\left(A_1 \times r^{\lambda_0+\lambda_1-2}f_1\right)$$

Compatibility equation becomes in the form of ,

$$B\times\frac{\partial^4\tilde{\phi}_i^k}{\partial\theta^4} = -C\left(\frac{\partial^3\tilde{\phi}_i^k}{\partial\theta^3}, \frac{\partial^2\tilde{\phi}_i^k}{\partial\theta^2}, \frac{\partial\tilde{\phi}_i^k}{\partial\theta}, \tilde{\phi}_i^k, n, \lambda_0\right) \tag{1}$$

Equation (1) is the fourth-order ordinary differential equation. The governing differential equation and boundary conditions define an eigenvalue problem. A fourth-order Runge-Kutta method and the shooting method were used to solve the problem.

3. RESULTS:

In the zero-th order approximation, from the solution of differential equation of the power-law hardening material side the singular exponent, λ_0 is calculated for different power-law hardening exponent, *n*. Displacement of Elastic material side from the zero-th order approximation is applied as the forced displacement to the power-law hardening material side in the first order approximation. Due to the forced displacement on the interface, displacement of first order approximation in the power law material side should be the same as the displacement of zero-th order approximation in the elastic material side.

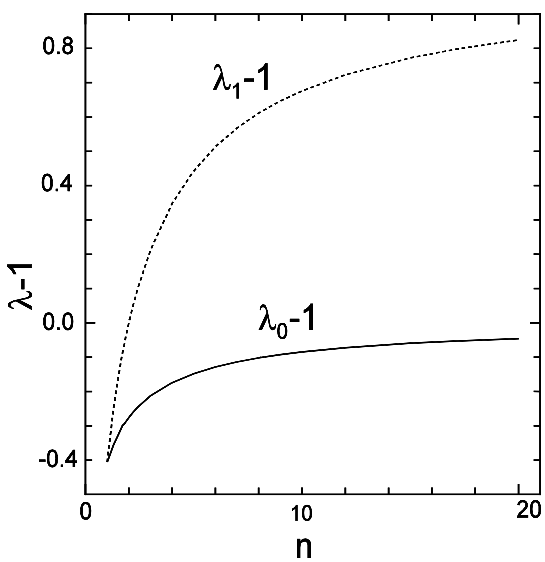

Figure 1:Graphical form of stress singularity $\lambda_i - 1$with n.

The iterative boundary condition can be expressed on the interface as, $\left(u_{r1}^{I}\right)_{\theta=0}=\left(u_{r0}^{II}\right)_{\theta=0}$ and $\left(u_{\theta1}^{I}\right)_{\theta=0}=\left(u_{\theta0}^{II}\right)_{\theta=0}$, where u_{r0}^{II} and $u_{\theta0}^{II}$ are the displacements of zero-th order approximation in the elastic material side, u_{r1}^{I} and $u_{\theta1}^{I}$ are the displacements of the first order approximation in the power-law hardening material side. $A_0^{n-1}A_1 r^{(n\lambda_0-n-\lambda_0+\lambda_1)+1}\left(\tilde{u}_{r1}^{I}\right)_{\theta=0}=A_0 r^{\lambda_0}\left(\tilde{u}_{r0}^{II}\right)_{\theta=0}$ and $A_0^{n-1}A_1 r^{(n\lambda_0-n-\lambda_0+\lambda_1)+1}\left(\tilde{u}_{\theta1}^{I}\right)_{\theta=0}=A_0 r^{\lambda_0}\left(\tilde{u}_{\theta0}^{II}\right)_{\theta=0}$, where $\tilde{u}_r$ and $\tilde{u}_\theta$ are the angular function terms.

To satisfy the boundary condition on the interface the power of *r* should be equal. Equating the power of r we have, $\lambda_0=n\lambda_0-n-\lambda_0+\lambda_1+1$ or $\lambda_1=\lambda_0+(1-n)(\lambda_0-1)$. Similarly, to satisfy the boundary condition on the interface in the higher order approximation the singular exponent can be expressed as: $\lambda_i=\lambda_0+i\times(1-n)(\lambda_0-1)$. It seems the i-th order singularity is depends on hardening exponent n and zero-th order singularity λ_0 .

Figure 1 shows the relation between the order of singularity, $\lambda-1$ and power law hardening exponent, n. As the hardening exponent in the power law hardening material is increased the order of the singularity tends to increase which means the absolute value of the order of singularity $|\lambda-1|$ tends to decrease.

4. CONCLUSIONS:

Singurar fields on the interface edge of elastic and power-law hardening materials joint were studied with different strain hardening exponent.The power of *r* in the stress equation depends on the hardening exponent *n*. Two stress singular terms exist in the first order approximation for n<2.0. The power of the first order singular term, $|\lambda_1-1|$, decreased with increasing *n*.

5. REFERENCES:

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